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# Multiple M2-branes and the embedding tensor

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**Abstract**

We show that the Bagger–Lambert theory of multiple M2-branes fits into the general construction of maximally supersymmetric gauge theories using the embedding tensor technique. We apply the embedding tensor technique in order to systematically obtain the consistent gaugings of $\mathcal{N} = 8$ superconformal theories in 2 + 1 dimensions. This leads to the Bagger–Lambert theory, with the embedding tensor playing the role of the four-index antisymmetric tensor defining a '3-algebra'. We present an alternative formulation of the theory in which the embedding tensor is replaced by a set of unrestricted scalar fields. By taking these scalar fields to be parity-odd, the Chern–Simons term can be made parity-invariant.

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## 1. Introduction

Recently, a three-dimensional world-volume theory describing a set of multiple M2-branes was proposed [1–3] (see also [4, 5]). The theory is based on the existence of a '3-algebra' that generalizes the Lie algebras of ordinary gauge theories to a structure involving an antisymmetric triple bracket

$$[T^a, T^b, T^c] = f^{abc}{}_d T^d, \tag{1.1}$$

where $T^a, a = 1, \ldots, N$, denote the generators of the algebra. Assuming the existence of a symmetric tensor $h_{ab}$ (which we will later take to be the Kronecker delta $\delta_{ab}$) to raise and lower indices, the generalized structure constants $f^{abcd}$ need to be totally antisymmetric,

$$f^{abcd} = f^{[abcd]}. \tag{1.2}$$

Apart from this linear identity, there is a quadratic identity (the so-called fundamental identity)[1],

$$f^{abe}{}_g f^{cdg}{}_f - f^{cde}{}_g f^{abg}{}_f - f^{abc}{}_g f^{dge}{}_f + f^{abd}{}_g f^{cge}{}_f = 0, \tag{1.3}$$

[1] Such structures also occur in the study of maximally supersymmetric solutions of supergravity theories [6].

which is the analogue of the Jacobi identity for Lie algebras. So far, only one explicit solution of constraints (1.2) and (1.3) is known, namely $f^{abcd} = \varepsilon^{abcd}$ for $N = 4$. In this case, it has been shown that the theory can be reinterpreted as an ordinary gauge theory based on the gauge group $SO(4) = SU(2) \times SU(2)$ [7].

The world-volume theory describing multiple M2-branes contains a set of embedding scalars $X_a^I, I = 1, \ldots, 8$ and a set of fermions $\Psi_a$. Furthermore, there is a set of world-volume gauge fields $A_{\mu ab} = -A_{\mu ba}$. A crucial feature of the theory is that these gauge fields do not describe independent degrees of freedom. They occur via a Chern–Simons term such that their field equations lead to a duality relation with the embedding scalars. This Chern–Simons term was introduced in an earlier attempt to construct a supersymmetric world-volume theory with 16 supercharges [8]. A nice feature of the Bagger–Lambert theory is that it reproduces the so-called Basu–Harvey equation [9] which was the original motivation for the proposal of [1–3].

The above two features, (i) a tensor that satisfies a linear and a quadratic constraint and (ii) gauge fields that occur via a Chern–Simons term, are very reminiscent of the so-called embedding tensor technique for constructing matter-coupled gauged supergravities. This method was originally proposed to construct maximal gauged supergravities in three dimensions [10, 11] and later applied to other cases in three dimensions [12–14]. The embedding tensor $\Theta$ plays the role of the tensor $f$ above and is used to specify which gauge fields are needed to gauge which subgroup of the duality group.

The case of matter-coupled half-maximal supergravity in three dimensions, with duality group $SO(8, N)$, was studied in [12–14]. The relevant embedding tensor is a four-index tensor $\Theta_{ab,cd}$ satisfying certain linear and quadratic constraints. A particular solution to these constraints is given by $\Theta_{ab,cd} = f_{abcd}$ satisfying (1.2) and (1.3). However, in supergravity there are more possibilities than this totally antisymmetric combination. Specifically, $\Theta$ can have a singlet (corresponding to a gauging of the full duality group) and a symmetric traceless part.

So far, the embedding tensor technique has been mainly applied to construct gauged supergravity theories but it can be used to construct supersymmetric gauge theories as well. For instance, it has been used to construct $\mathcal{N} = 2$, $D = 4$ supersymmetric gauge theories with electric and magnetic charges [15]. In this communication we wish to apply the embedding tensor technique to the case of $\mathcal{N} = 8$ supersymmetric gauge theories in three dimensions and find out whether generalizations of the Bagger–Lambert model are possible or not. This investigation is also a nice illustration of how the embedding tensor technique works in general.

## 2. Gauging $\mathcal{N} = 8$ superconformal theories

Our starting point is the free superconformal $\mathcal{N} = 8$ theory in $D = 3$ with $N$ matter multiplets, i.e. containing $8N$ scalars $X^{aI}$ and $8N$ Majorana spinors $\Psi^{aA}$. Here and in the following $I, J = 1, \ldots, 8$, $A, B = 1, \ldots, 8$ and $\dot{A}, \dot{B} = 1, \ldots, 8$ denote, respectively, the vector, spinor and conjugate spinor indices of the $SO(8)$ R-symmetry group. The theory is described by the Lagrangian

$$\mathcal{L} = -\frac{1}{2}\partial^\mu X^{aI}\partial_\mu X_a^I + \frac{\mathrm{i}}{2}\bar{\Psi}^{aA}\Gamma^\mu\partial_\mu\Psi_a^A. \tag{2.1}$$

It is invariant under the supersymmetry transformations[2] (with $\Gamma_{012}\epsilon = \epsilon$)

[2] We suppress the $SO(8)$ spinor indices whenever they are not explicitly required.

$$\delta_\epsilon X^I_a = \mathrm{i}\bar{\epsilon}\Gamma^I\Psi_a, \qquad \delta_\epsilon\Psi_a = \partial_\mu X^I_a\Gamma^\mu\Gamma^I\epsilon, \tag{2.2}$$

and under the global symmetry group $SO(8)\times SO(N)$.

We wish to analyze the question of which subgroups of this global symmetry group can be promoted to a local symmetry. Since in this communication our ultimate motivation is the M2 brane example, we will restrict ourselves to gauge groups that lie inside the $SO(N)$ factor. As usual, we first introduce gauge-covariant derivatives that couple the scalars to the gauge vectors. As in [10, 12] these gauge fields come in the adjoint representation of the global symmetry group $G_{\text{global}}$ and enter only via a Chern–Simons term. The covariant derivatives read

$$D_\mu = \partial_\mu - g\Theta_{\alpha\beta}A_\mu{}^\alpha t^\beta, \tag{2.3}$$

where $g$ is the gauge coupling constant and the indices $\alpha,\beta = 1,\ldots,\dim G_{\text{global}}$ label the adjoint of the rigid symmetry group, spanned by the generators $t^\alpha$ with structure constants $f^{\alpha\beta}{}_\gamma$. The symmetric embedding tensor $\Theta_{\alpha\beta} = \Theta_{\beta\alpha}$ encodes the embedding of the gauge group $G_0$ into $G_{\text{global}}$ in that $G_0$ is spanned by generators

$$X_\alpha = \Theta_{\alpha\beta}t^\beta. \tag{2.4}$$

In other words, $\Theta$ acts as a projector which singles out those generators that participate in the gauging. Gauge invariance of the theory to be constructed requires invariance of $\Theta_{\alpha\beta}$ under the adjoint action of the gauge group generators $X_\alpha$. This implies the quadratic constraint

$$\mathcal{Q}_{\alpha,\beta\gamma} \equiv \Theta_{\alpha\epsilon}\Theta_{\delta(\beta}f^{\delta\epsilon}{}_{\gamma)} = 0, \tag{2.5}$$

which also ensures the closure of the gauge algebra spanned by (2.4).

In the case at hand, the indices split according to the adjoint of $SO(N)$, i.e. $\alpha = [ab]$. Consequently, the embedding tensor reads $\Theta_{ab,cd}$ and has the symmetries

$$\Theta_{ab,cd} = -\Theta_{ba,cd} = -\Theta_{ab,dc} = \Theta_{cd,ab}. \tag{2.6}$$

Using the explicit form of the structure constants

$$f^{ab,cd,}{}_{ef} = -2\delta^{[a}{}_{[e}\delta^{b][c}\delta^{d]}{}_{f]}, \tag{2.7}$$

the quadratic constraint reads

$$\mathcal{Q}_{ab,cd,ef} \equiv \Theta_{ab,e}{}^g\Theta_{cd,gf} - \Theta_{cd,e}{}^g\Theta_{ab,gf} - \Theta_{ab,c}{}^g\Theta_{dg,ef} + \Theta_{ab,d}{}^g\Theta_{cg,ef} = 0. \tag{2.8}$$

Moreover, the generators in (2.3) act in the fundamental representation, $(t^{ab})^c{}_d = \delta^{[a}{}_d\delta^{b]c}$, i.e. the explicit form of the covariant derivative is given by

$$D_\mu X^I_d = \partial_\mu X^I_d - gA_\mu{}^{ab}\Theta_{ab,cd}X^{cI}, \tag{2.9}$$

and similarly for the spinors.

Let us now turn to the gauged action. Our starting point is the following ansatz:

$$\begin{aligned}\mathcal{L}_g = &-\frac{1}{2}D^\mu X^{aI}D_\mu X^I_a + \frac{\mathrm{i}}{2}\bar{\Psi}^a\Gamma^\mu D_\mu\Psi_a + \frac{\mathrm{i}}{4}g\bar{\Psi}^{aA}A_{3\,aA,bB}(X)\Psi^{bB}\\ &+\frac{1}{2}g\varepsilon^{\mu\nu\lambda}A_\mu{}^\alpha\Theta_{\alpha\beta}\left(\partial_\nu A_\lambda{}^\beta - \frac{1}{3}g\Theta_{\gamma\delta}f^{\beta\delta}{}_\epsilon A_\nu{}^\gamma A_\lambda{}^\epsilon\right) - g^2V(X).\end{aligned} \tag{2.10}$$

As in supergravity, we added Yukawa-like couplings parametrized by a scalar-dependent function $A_3(X)$ as well as a scalar potential $V(X)$ and a Chern–Simons term. By virtue of the quadratic constraint (2.5), this action is gauge-invariant under

$$\begin{aligned}\delta X^I_a &= -g\Lambda^\alpha\Theta_{\alpha\beta}(t^\beta)^b{}_aX^I_b = g\Lambda^{cd}\Theta_{cd,ba}X^{bI},\\ \delta\Psi_a &= g\Lambda^{cd}\Theta_{cd,ba}\Psi^b,\\ \delta A_\mu{}^\alpha &= D_\mu\Lambda^\alpha = \partial_\mu\Lambda^\alpha - g\Theta_{\beta\gamma}f^{\alpha\beta}{}_\delta A_\mu{}^\gamma\Lambda^\delta.\end{aligned} \tag{2.11}$$

For the gauge vectors ${A_\mu}^{ab}$ with explicit $SO(N)$ indices, the gauge variation can be rewritten by the use of the structure constants (2.7) as

$$\delta A_{\mu\, ab} = \partial_\mu \Lambda_{ab} + g\Theta_{ae,cd}{A_\mu}^{cd}{\Lambda^e}_b - g\Theta_{be,cd}{A_\mu}^{cd}{\Lambda^e}_a. \tag{2.12}$$

Next, we are going to analyze the question for which choices of the embedding tensor the action corresponding to (2.10) can be made supersymmetric. We use the following ansatz for the supersymmetry variation of the fermions

$$\delta_\epsilon \Psi_{aA} = D_\mu X^I_a \Gamma^\mu \Gamma^I_{A\dot{B}}\epsilon^{\dot{B}} + gA_{2\,aA\dot{B}}(X)\epsilon^{\dot{B}}, \tag{2.13}$$

where we introduced a gauge-covariant derivative and added a scalar-dependent fermion shift function $A_2(X)$. Due to the non-commutativity of covariant derivatives, the supersymmetry variation of the kinetic terms in (2.10) no longer vanishes, but instead gives rise to a term proportional to the field strength,

$$\delta_\epsilon \mathcal{L}_{\text{kin}} = \frac{\mathrm{i}}{2} g\Theta_{ab,cd}\bar{\Psi}^a\Gamma^{\mu\nu}\Gamma^I\epsilon {F_{\mu\nu}}^{cd}X^{bI}. \tag{2.14}$$

These can be compensated by assigning a non-trivial supersymmetry variation to the gauge vectors,

$$\delta_\epsilon {A_\mu}^{ab} = \mathrm{i}\bar{\epsilon}\Gamma_\mu\Gamma^I X^{I[a}\Psi^{b]}, \tag{2.15}$$

such that the variation of the Chern–Simons term precisely cancels (2.14). However, these variations of the gauge field give rise to additional variations from its presence inside the covariant derivatives, and the problem is to determine $A_2(X)$ and $A_3(X)$ such that these contributions can be canceled.

In the embedding tensor formalism this problem of finding a consistent supersymmetric deformation translates into the problem of finding the right linear constraints on the embedding tensor. *A priori*, $\Theta_{ab,cd}$ with symmetries (2.6) takes values in the symmetric tensor product

$$\left(\yng(1,1)\otimes\yng(1,1)\right)_{\text{sym}} = \mathbf{1}\oplus\yng(1,1,1,1)\oplus\yng(2,2)\oplus\yng(2)\,. \tag{2.16}$$

This corresponds to the general parametrization

$$\Theta_{ab,cd} = f\delta_{c[a}\delta_{b]d} + f_{abcd} + f^{(2,2)}_{ac,bd} + f_{[c[a}\delta_{b]d]}, \tag{2.17}$$

where $f_{abcd} = f_{[abcd]}$ denotes the totally antisymmetric part and $f^{(2,2)}_{ac,bd}$ has the window symmetries of $\yng(2,2)$, in particular $f^{(2,2)}_{[ac,b]d} = 0$. Supersymmetry then implies that only some of these irreducible representations are consistent, or in other words, beyond the quadratic constraint (2.8) it requires a $G_{\text{global}}$-covariant linear constraint.

To determine these constraints, we first focus on the variations linear in the gauge coupling $g$ (and thus $\Theta$) and linear in the fermions $\Psi$. The variation $\delta {A_\mu}^{ab}$ inside the covariant derivative on the scalars gives rise to terms of the form $X^2 DX\Psi$. In order to cancel these it follows that $A_3(X)$ has to be quadratic in $X$ and, consequently, $A_2(X)$ has to be cubic in $X$. The most general ansatz in terms of the embedding tensor reads

$$\begin{aligned} A_{3\,aA,bB} &= b_1\Theta_{ab,cd}\Gamma^{IJ}_{AB}X^I_cX^J_d + b_2\Theta_{ac,bd}\delta_{AB}X^I_cX^I_d + b_3\Theta_{ac,bd}\delta^{cd}\delta_{AB}X^I_eX^I_e, \\ A_{2\,aA\dot{B}} &= \Theta_{ab,cd}\left(c_1\Gamma^{IJK}_{A\dot{B}}X^{Ib}X^{Jc}X^{Kd} + c_2\Gamma^I_{A\dot{B}}X^{Id}X^{Jb}X^{Jc}\right). \end{aligned} \tag{2.18}$$

The variation of the Lagrangian gives rise to a term proportional to $b_1\Gamma^{IJ}\Gamma^K$, containing the antisymmetric part $\Gamma^{IJK}$. These have to be canceled by choosing the coefficient $c_1$ in $A_2$ and thus in the supersymmetry variation of the fermion in the right way. However, from (2.18) one infers that this term in $A_2$ can only be non-zero if $\Theta_{[ab,c]d}$ is non-zero. This, in turn, implies that only the totally antisymmetric $f_{abcd}$ in (2.17) can give rise to a consistent gauging. Specifically one finds

$$b_1 = -1, \qquad b_2 = b_3 = 0, \qquad c_1 = \tfrac{1}{6}, \qquad c_2 = 0 \tag{2.19}$$

and the following expression for the scalar potential $V(X)$:

$$V(X) = \tfrac{1}{12}\Theta_{ab,cg}\Theta_{de,f}{}^{g} X^{aI} X^{bJ} X^{cK} X^{dI} X^{eJ} X^{fK}. \tag{2.20}$$

In total, the linear constraint imposed by supersymmetry reads

$$\left(\mathbb{P}_{\mathbf{1}} \oplus \begin{array}{|c|c|}\hline & \\ \hline & \\ \hline \end{array} \oplus \begin{array}{|c|c|}\hline & \\ \hline \end{array}\right) \Theta_{ab,cd} = 0 , \tag{2.21}$$

where $\mathbb{P}$ projects out those representations that are not totally antisymmetric. A particular solution, for $N = 4$, is given by $\Theta_{ab,cd} = \varepsilon_{abcd}$, which is an invariant tensor of $SO(4)$ and solves the quadratic constraints (2.8). This leads to the $SO(4)$ gauge theory example of [1–3].

The general solution $\Theta_{ab,cd} = f_{abcd}$ of (2.21) gives back the Bagger–Lambert theory. In fact, for a totally antisymmetric embedding tensor the quadratic constraint (2.8) precisely reduces to the fundamental identity (1.3). Moreover, all the couplings match. In particular, the Chern–Simons term of [2] based on the 3-algebra structure constants precisely coincides with the Chern–Simons action in (2.10), as can be checked by the insertion of (2.7).

In order to illustrate the use of the embedding tensor, let us briefly comment on different choices of gauge groups and their embedding. First of all, for any $N \geqslant 4$ the linear constraint (2.21) allows for a consistent gauging of the subgroup $SO(4) \subset SO(N)$. Splitting the indices as $a = (i, 5, \ldots, N)$, this corresponds to a choice of embedding tensor, in which the only non-vanishing components are

$$\Theta_{ij,kl} = \varepsilon_{ijkl}. \tag{2.22}$$

This is an invariant tensor of the subgroup to be gauged, and it solves the quadratic constraint. Depending on the value of N, larger gauge groups may be possible. For instance, for $N = 8$ the canonically embedded $SO(4) \times SO(4)$ subgroup can be gauged, corresponding to an embedding tensor of the form

$$\Theta_{ij,kl} = \kappa_1 \varepsilon_{ijkl}, \qquad \Theta_{\bar{i}\bar{j},\bar{k}\bar{l}} = \kappa_2 \varepsilon_{\bar{i}\bar{j}\bar{k}\bar{l}}. \tag{2.23}$$

Here we have split the indices according to $a = (i, \bar{i}) = (1, \ldots, 4, \bar{1}, \ldots, \bar{4})$ and introduced two arbitrary coupling constants $\kappa_1, \kappa_2$.

One may wonder whether more interesting gauge groups are possible, beyond the various copies of $SO(4)$. An attractive candidate is $G_2$ in case of $N \geqslant 7$. In fact, it can be defined as the subgroup of $SO(7)$ that leaves a certain antisymmetric 4-tensor $C_{abcd}$ invariant, and so one may take $\Theta_{ab,cd} = f_{abcd} = C_{abcd}$. (For a concise account of $G_2$ see, for instance, appendix A of [16].) This ansatz has been pursued in [17] (see also [18]), with negative results. The fundamental identity (1.3) is not satisfied and so $G_2$ does not give rise to a consistent 3-algebra. It is, however, instructive to reexamine this problem from the point of view of the embedding tensor formalism. From this perspective, the embedding tensor should act as a projector from $SO(7)$ onto $G_2$ according to (2.4). It is possible to find such a projector which is totally antisymmetric, i.e., satisfying the linear constraint, and which gives rise to the closed $G_2$ algebra. However, this tensor is not $G_2$ invariant and therefore the quadratic constraints (2.8) are still not satisfied. Instead one may start from the 4-tensor of $G_2$ which is known to be invariant. It is given by

$$C_{abcd} = \tfrac{1}{3!}\varepsilon^{abcdefg}\hat{C}_{efg}, \tag{2.24}$$

where $\hat{C}_{abc}$ is defined by

$$\hat{C}_{ijk} = \varepsilon_{ijk}, \qquad \hat{C}_{i\bar{j}\bar{k}} = \hat{C}_{\bar{i}j\bar{k}} = \hat{C}_{\bar{i}\bar{j}k} = -\varepsilon_{ijk}, \qquad \hat{C}_{7i\bar{j}} = \delta_{ij}, \tag{2.25}$$

and we have split the indices according to $a = (i, \bar{i}, 7)$. However, (2.24) is not a projector onto $G_2$. A possible solution for the embedding tensor, respectively the projector, is instead given by

$$\Theta_{ab,cd} = \delta_{c[a}\delta_{b]d} + \tfrac{1}{4}C_{abcd}. \tag{2.26}$$

By the insertion of (2.26) into (2.8) one can verify that this indeed solves the quadratic constraints. But it does not solve the linear constraint (2.21) due to the presence of a singlet combination. However, as we will discuss in the following section, in gauged supergravity these singlet components are allowed [11, 13]. Therefore, we conclude that while the superconformal theories do not allow for a gauging of $G_2$, the embedding tensor (2.26) does give rise to a consistently gauged $\mathcal{N} = 8$ supergravity. To the best of our knowledge this example has not appeared in the literature before.

## 3. Comparing with gauged supergravity

In this section we will compare the application of the embedding tensor technique to both half-maximal matter-coupled gauged supergravities as well as to maximal supersymmetric gauge theories. Both theories have an equal number of supercharges. In this section we will not only consider $D = 3$ but also $3 < D \leqslant 10$ dimensions. It turns out that the supergravity theories allow for more consistent gaugings. This is due to the fact that these theories have a weaker linear constraint and a less trivial (non-compact) duality group to start with.

It is of interest to compare the embedding tensors corresponding to the supergravity and gauge theory cases in more detail. At first sight the two gaugings are unrelated since the relevant scalar manifolds differ. For instance, the $D = 3$ supergravity case leads to the scalar manifolds $SO(8, N)/SO(8) \times SO(N)$, whereas in the $D = 3$ gauge theory case one deals with the flat manifolds $\mathbb{R}^{8N}$. Nevertheless, we will argue below that the embedding tensor representations for the gauge theories can be deduced from the corresponding supergravity representations. For the supergravity case the representations of the embedding tensors have been calculated [12–14, 19], see table 1.[3] We see that for supergravity in $6 \leqslant D \leqslant 10$ dimensions there is a fundamental and three-form representation. In these dimensions the fundamental representation corresponds to the gauging of a $SO(1, 1)$ diagonal subgroup of the $SO(1, 1) \times SO(10-D, 10-D+N)$ duality group[4]. The three-form representation represents the antisymmetric structure constants of the subgroup $G_0 \subset SO(10 - D, 10 - D + N)$ that is gauged. The reason that the fundamental representation is absent in the gauge theory case is that the corresponding $SO(1, 1)$ symmetry that is gauged involves a shift of the dilaton supergravity field, which is absent in the gauge theory.

To be able to do more general gaugings one needs more spacetime vectors than only the fundamental representation, which is present in all these dimensions. For example, in $D = 5$ an additional vector is provided by the dual of the NS–NS two-form, giving rise to an extra two-form representation of possible gaugings. Since this extra possibility is due to the dualization of a supergravity field, this extra two-form representation is absent in the gauge theory. In $D = 4$ the extra vectors are the Hodge duals of the original ones, leading to an $SL(2, \mathbb{R})$ doublet of possible gaugings. This possibility arises both in the supergravity

[3] We have indicated in the table only gaugings, no massive deformations. Furthermore, we have ignored the chiral case in $D = 6$ dimensions. For more details, see table 3 in [19].

[4] Actually, the quadratic constraints forbid this gauging in $D = 10$ because $SO(N)$ has no $SO(1, 1)$ subgroup.

**Table 1.** This table gives, for $3 \leqslant D \leqslant 10$, the duality group representations of the embedding tensors for matter-coupled half-maximal gauged supergravity (second column) and maximal supersymmetric gauge theory (third column). In $D = 4$ the **2** refers to the fundamental representation of the electro-magnetic $SL(2, \mathbb{R})$ duality.

| $D$ | Supergravity | Gauge theory |
|---|---|---|
| 10 – 6 | $\square$ $\quad$ $\yng(1,1,1)$ | $\yng(1,1,1)$ |
| 5 | $\square$ $\quad$ $\yng(1,1)$ $\quad$ $\yng(1,1,1)$ | $\yng(1,1,1)$ |
| 4 | $(\mathbf{2}, \square)$ $\quad$ $\left(\mathbf{2}, \yng(1,1,1)\right)$ | $\left(\mathbf{2}, \yng(1,1,1)\right)$ |
| 3 | 1 $\quad$ $\yng(2)$ $\quad$ $\yng(1,1,1,1)$ | $\yng(1,1,1,1)$ |

and in the gauge theory case. In the gauge theory case, these correspond to the electric and magnetic gaugings discussed, for $\mathcal{N} = 2$ supersymmetry, in [15]. Finally, in $D = 3$ further gaugings become available due to the fact that scalars become dual to vectors. Note that the singlet and symmetric traceless representation, present in supergravity, have their origin in the four-dimensional $(\mathbf{2}, \square)$ representation which is absent in the gauge theory. That is why the 1 and $\yng(2)$ representations are absent in the $D = 3$ gauge theory.

Summarizing, by comparing with half-maximal supergravity we obtain a natural prediction for the embedding tensors of maximal supersymmetric gauge theories as presented in table 1. This includes the four-index antisymmetric representation of [1–3].

## 4. An alternative formulation

So far, we have introduced a Lagrangian containing a constant embedding tensor $\Theta_{ab,cd}$ satisfying the linear constraint (2.21) and the quadratic constraints (2.8). Even though in this formalism the gauging takes a completely covariant form with respect to $G_{\text{global}}$, this group is no longer an invariance of the Lagrangian. In fact, the $\Theta_{ab,cd}$ are not dynamical objects and therefore cannot transform under the symmetry. Instead, following [20, 21], we can promote the embedding tensor to a set of unconstrained scalar fields $\Theta_{ab,cd}(x)$ with the same symmetry properties by introducing two kinds of Lagrange multipliers. The first set consists of two-form potentials $B_{\mu\nu}{}^{ab,cd}$ with the same symmetry properties as $\Theta$. These can be viewed as the duals of $\Theta$ (see below) and their field equations will impose the constancy of $\Theta$. The second set consists of three-form potentials $C_{\mu\nu\rho}{}^{ab,cd,ef}$ which will impose the quadratic constraints by their equations of motion. They have the same symmetry properties as the quadratic constraint tensor $\mathcal{Q}_{ab,cd,ef}$ defined in (2.8). The total Lagrangian is then given by

$$\mathcal{L}_{\text{total}} = \mathcal{L}_g - \tfrac{1}{2} g \varepsilon^{\mu\nu\rho} \partial_\mu \Theta_{ab,cd} B_{\nu\rho}{}^{ab,cd} - \tfrac{1}{3} g^2 \varepsilon^{\mu\nu\rho} \mathcal{Q}_{ab,cd,ef} C_{\mu\nu\rho}{}^{ab,cd,ef}, \tag{4.1}$$

where we replaced in $\mathcal{L}_g$, see equation (2.10), everywhere $\Theta$ by the spacetime-dependent $\Theta(x)$.

The Lagrangian $\mathcal{L}_g$ is not gauge-invariant, neither is its action supersymmetric[5]. However, the violation of these symmetries is proportional to $\partial\Theta$ or to the quadratic constraint tensor

[5] Of course, we should also in the transformation rules replace $\Theta$ by its spacetime-dependent form.

$\mathcal{Q}$. Such terms can always be canceled by assigning appropriate gauge transformations and supersymmetries to the Lagrange multipliers $B$ and $C$. To illustrate this, we give the full bosonic gauge transformations [21], for which we find

$$\begin{aligned}
\delta_\Lambda B_{\mu\nu}{}^{ab,cd} &= \partial_{[\mu}\Lambda_{\nu]}{}^{ab,cd} + D_{[\mu}\Lambda^{ab}A_{\nu]}{}^{cd} - 2\varepsilon_{\mu\nu\rho}\Lambda^{ab}X^{Ic}D^\rho X^{dI} \\
&\quad + \mathrm{i}\Lambda^{ab}\bar{\Psi}^c\Gamma_{\mu\nu}\Psi^d + \tfrac{8}{3}g\left(\Theta_{ef,g}{}^d\Lambda_{\mu\nu}{}^{ab,ef,cg} - \Theta_{ef,g}{}^c\Lambda_{\mu\nu}{}^{ef,ab,gd}\right), \\
\delta_\Lambda C_{\mu\nu\rho}{}^{ab,cd,ef} &= \partial_{[\mu}\Lambda_{\nu\rho]}{}^{ab,cd,ef} - 2A_{[\mu}{}^{ab}A_\nu{}^{cd}D_{\rho]}\Lambda^{ef} - \varepsilon_{\mu\nu\rho}A_\sigma{}^{ab}\Lambda^{cd}X^{eI}D^\sigma X^{fI} \\
&\quad + 3\mathrm{i}A_{[\mu}{}^{ab}\bar{\Psi}^c\Gamma_{\nu\rho]}\Psi^d\Lambda^{ef} + \tfrac{1}{8}\mathrm{i}\varepsilon_{\mu\nu\rho}\Lambda^{ab}\bar{\Psi}^c\Gamma^{IJ}\Psi^d X^{eI}X^{fJ} \\
&\quad + \tfrac{1}{24}g\Theta_{gh,i}{}^f\varepsilon_{\mu\nu\rho}\Lambda^{ab}X^{Ic}X^{Jd}X^{Ke}X^{Ig}X^{Jh}X^{Ki} \\
&\quad - \tfrac{1}{24}g\Theta_{gh,i}{}^f\varepsilon_{\mu\nu\rho}\Lambda^{ab}X^{Ig}X^{Jh}X^{Ki}X^{Ic}X^{Jd}X^{Ke}.
\end{aligned} \tag{4.2}$$

Here we have left implicit the Young projection of the right-hand sides according to the symmetries of the left-hand sides. We will not give the supersymmetry transformations, since they are not very illuminating. We finally note that the field equations of the scalar fields $\Theta$ give rise to a duality relation between the two-form potentials $B$ and the embedding scalars $\Theta$ [20, 21].

## 5. Discussion

In this communication we have presented a derivation of the Bagger–Lambert theory of multiple M2-branes by an application of the embedding tensor method to $\mathcal{N} = 8$ supersymmetric gauge theories in three dimensions. The linear constraint imposed by global supersymmetry restricts the embedding tensor to an antisymmetric four-index tensor, giving rise to the Bagger–Lambert theory. This is in contrast to the case of $\mathcal{N} = 8$ supergravity, where the linear constraint also allows for a symmetric traceless tensor and a singlet [13]. These representations lead to extra gaugings in the supergravity case. For instance, for $f_{abcd} = 0$ a consistent gauging is obtained by the embedding of the compact gauge group $SO(p) \times SO(N-p)$ into $SO(N)$ with opposite coupling constant for the two different groups [13]. We hope that the relation with gauged supergravities can be helpful in finding more solutions to the quadratic constraints.

In a second stage we have replaced the embedding tensor $\Theta$ by scalar fields $\Theta(x)$. This has several advantages. First of all the Chern–Simons terms can now be made manifestly invariant under parity transformations by taking the scalars $\Theta$ to be odd under parity. Second, the theory contains less free parameters: the constants $\Theta$ have become integration constants that occur only after solving the equations of motion. Third, the scalars $\Theta$ allow the possibility of domain walls on the M2-brane world-volume where, upon crossing the domain wall, the constants $\Theta$ change value [22].

It is of interest to search for generalizations of the Bagger–Lambert model. For a recent discussion, see [23]. Another promising approach is to consider supersymmetric gauge theories without a Lagrangian [24]. In fact, gauged supergravities without a Lagrangian have already been considered in the literature, see, e.g. [25]. We expect that the application of the embedding tensor technique in these cases will lead to more general gaugings.

Quite a few papers have appeared recently addressing different issues concerning the world-volume theory of multiple M2-branes. In particular, the relation with multiple D2-branes has been clarified [26] (see also [24]), the $OSp(8|4)$ superconformal symmetry of the model has been verified [17], the boundary theory of open membranes has been considered [27] and it has been shown that the $SO(4)$ gauge theory solution corresponds to two M2-branes moving on a non-trivial manifold [28, 29]. We hope that this note will help in further clarifying the relation between (and possible extensions of) $\mathcal{N} = 8$ superconformal theories and multiple M2-branes.

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